BARUCH COLLEGE

MATH 2205

FALL 2007

MANUAL FOR THE UNIFORM FINAL EXAMINATION

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The final examination for Math 2205 will consist of two parts.

Part I: This part will consist of 25 questions similar to the questions that appear in Part I of each sample exam.
No calculator will be allowed on this part.

Part II: This part will consist of 10 questions similar to the questions that appear in Part II of each sample exam.
The graphing calculator is allowed on this part.

There may be a few new problem types on the exam that are not similar to the problems in the sample exams. If such problems appear, they will be similar to problems that you have seen during the semester.

GRADING: Each question will be worth 3 points.
Anyone who gets 34 or 35 questions correct will be assigned a grade of 100.
No points are subtracted for wrong answers.

CONTENTS OF THIS MANUAL:

Page showing the sample questions that correspond to each section of the current text.
(When a section has been covered in class, the list indicates the problems that can be used in studying for the exam that includes that section during the semester.)
TI-89 Facts for the Uniform Final Examination.
(This portion indicates the minimal calculator knowledge needed.)
Sample Exam A (A1 – A35)
Sample Exam B (B1 – B35)
Sample Exam C (C1 – C35)
Sample Exam D (D1 – D35)
Sample Exam E (E1 – E35)
Answers to the problems.

Math 2205
Textbook Sections Corresponding to Sample Uniform Final Exam Questions
Fall 2007

Textbook: Applied Calculus, Gordon, Wang, Materowski, Baruch College, CUNY

Section	Problems
3.1 (Extrema)	A1, A19, B1, B19, B25, C1, C18, D1, D12, D19, D25, E1, E12, E13
	A26, B26, C26, D34
3.2 (1st Der. Test)	A2, B2, C2, D2, E2
3.3 (Concavity)	A3, A6, A20, B3, B20, C3, C19, D3, D20, E3, E4, E20, E23
	A27, A34, B27, C27, C34, D26, E34
3.4 (Geom. Apps.)	A4, B4, C4, C20, D4, E19
3.5 (Business Apps.)	A5, A21, B5, B21, C7, C21, D5, D14, D21, E5, E14, E21
	B28
3.6 (Linearization)	A22, B6, B22, C6, C22, D6, D15, D22, E6, E15
	A28, B29, C28, D27, D28, E35
4.1 (Inverses)	A7, B7, C5
4.2 (Exponent. F'cns)	A8, B8, C8, D13
	B32
4.3 (Number e)	A9, B9, C9
	A29, C29, E26, E27
4.4 (Derivative e^x)	A10, A23, B10, B23, C10, C23, D7, D16, E7
	A30, A33, B35, C30, D29, E28, E29
4.5 (Logarithms)	A11, B11, C11, E8, E16
	A35, D30, E30
4.6 (Log. Props/Der)	A12, A24, B12, B24, C12, C24, D8, D17, D23, E9, E17, E22, E25
	A31, B30, B34, C31, D31, D32, E31
4.7 (Applications)	B31, B33, C33, D35, E32
5.1 (Antiderivatives)	(See 5.2 Problems)
5.2 (Apps. Antider.)	A13, B13, C13, D9, E10
5.3 (Substitution)	A14, B14
5.4 (Approx. Area)	A15, B18
	A31, C32, E33
5.5 (Sigma and Area)	B15
5.6 (Definite Integral)	A16, B16, C15, D10
5.7 (Subst. Def. Int.)	A17, A25, B17, C14, C16, C25, D11, D18, E11, E18, E24
5.8 (Applications)	A18, C17
	A32, C35, D33

TI-89 FACTS FOR THE UNIFORM FINAL EXAMINATION

The following information represents the minimal calculator usage students should be familiar with when they take the uniform final examination. Instructors are expected to provide much more information in class.

Limits can be evaluated with the TI-89 by using the limit function whose syntax is

limit(*expression, variable, value*).

This function is found on the Home screen by using the F3 Calc key. The calculus menu that appears when F3 is pressed is shown in Figure 1. The

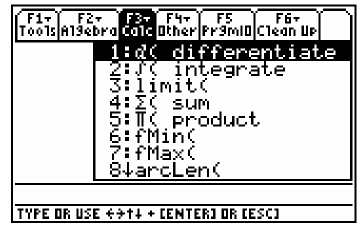


Figure 1

limit function is selected by either pressing 3 or using the down arrow cursor to highlight choice 3:limit and then pressing ENTER.

Example 1: Find $\lim_{x \to 16} \dfrac{x-4}{16-\sqrt{x}}$

Solution: Select the limit function as indicated above.
Complete the command line as follows and press ENTER:

limit((x-4)/(16-√(x)),x,16)

The answer is 1 as shown in Figure 2.

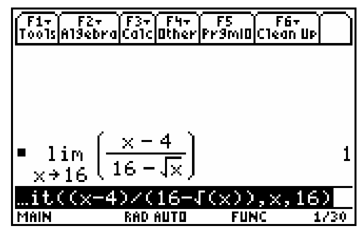


Figure 2

Example 2: Find $\lim_{\Delta x \to 0} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x}$

Solution: In order to use the calculator for this problem it is best to think of Δx as a single letter such as t. That is,

$$\lim_{t \to 0} \frac{\dfrac{1}{x+t} - \dfrac{1}{x}}{t}$$

Now enter on the command line the following:

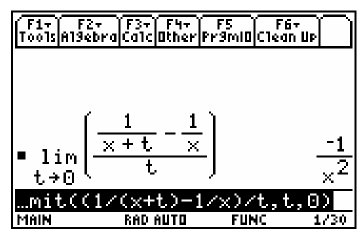


Figure 3

limit((1/(x+t)-1/x)/t,t,0) The result is $-1/x^2$ as shown in Figure 3.

Note that example 2 could have been done without a calculator if the limit requested was recognized as the definition of the derivative of f(x) = 1/x for which $f'(x) = -1/x^2$.

The syntax for the solve command and for finding the first or second derivative is:

solve(*equation, variable*)
***d*(*expression, variable*)** (first derivative, f ′(x), where f (x) = *expression*)
***d*(*expression, variable*, 2)** (second derivative, f ″(x), where f (x) = *expression*)

The solve command is obtained by pressing the F2 (Algebra menu) key in the Home Screen as shown in Figure 4 and then pressing ENTER to select choice 1:solve.

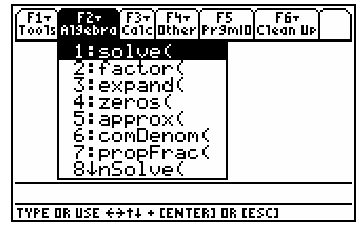


Figure 4

The differentiation operator *d* is choice 1 on the Home screen calculus menu shown in Figure 1. It can be more easily accessed by pressing the yellow 2nd key and then 8 (the *d* appears above the 8 in yellow). The purple D appearing above the comma key cannot be used for this purpose.

If an answer such as e^3 or 15/236 appears when a decimal answer is desired, the command can be repeated with the green diamond ◆ key pressed before pressing ENTER.

Example 3: The demand function for a product is p = 10 - ln(x), where x is the number of units of the product sold and p is the price in dollars. Find the value of x for which the marginal revenue is 0.

Solution: The revenue function is R(x) = px = x(10 - ln(x)).
The marginal revenue is the derivative, R′(x). This can be found by hand or by using the calculator command

d(x*(10-ln(x)),x)

obtained by pressing the keys

2nd 8 (x * (1 0 - 2nd x x)) , x) ENTER

as shown in Figure 5. The result is R′(x) = 9 - ln(x).

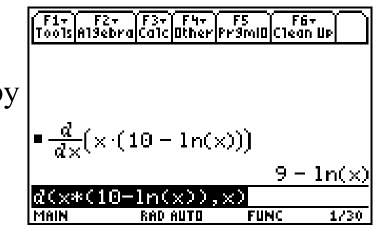


Figure 5

To find the value of x for which the marginal revenue is 0 the equation 9 - ln(x) = 0 must be solved.

Press F2 (Algebra) ENTER as shown in Figure 4. Then complete the command line as

solve(9-ln(x)=0,x)

Pressing ENTER produces the result e^9. Since a decimal answer is desired, repeat the command by first pressing the green diamond ◆ key before pressing ENTER. The final answer is 8103.08 as shown in Figure 6 and can be rounded off to 8103.

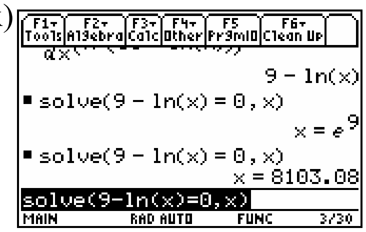


Figure 6

Example 4: The function $h(t) = \dfrac{30}{10 + e^{-0.1t+5}}$ has exactly one inflection point. Find it.

Solution: Recall that an inflection point occurs if h″(t) = 0 and the concavity changes sign. So first the second derivative is found with the command

d(30/(10+e^(-0.1t+5)),t,2)

where e^(is obtained by pressing ◆x
(e^x appears in green over the x key)

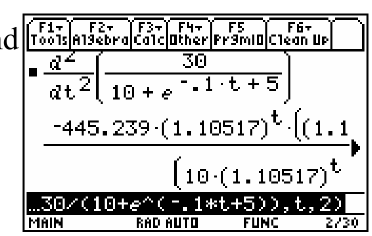


Figure 7

Figure 7 is the result. Recall that to see the entire result the up cursor arrow button must be pressed and then the right cursor arrow button must be pressed until the rest of the result on the right can be seen as shown in Figure 8. Thus,

$$h''(t) = \frac{-445.239(1.10517)^t((1.10517)^t - 14.8413)}{(10(1.10517)^t + e^5)^3}$$

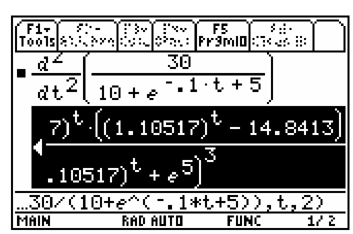


Figure 8

Move the cursor down to the command line again. The easiest way to solve h″(t) = 0 is as follows. First press F2 (Algebra) and select choice 1 so that the command line now only shows

solve(

Press the up cursor arrow to highlight the expression for h″(t) and press ENTER. The command line now shows the end of

solve(*above expression*

Complete the command by entering

=0,t)ENTER

The answer appears in Figure 9.

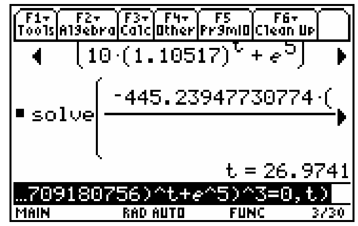


Figure 9

Since only one answer appears and it is known that an inflection point exists, there is no need to check to see if the concavity changes at t = 26.9741. It only remains to find the value of the original function at t = 26.9741. To do so, enter

30/(10+e^(-0.1*26.9741+5))

on the command line to obtain h(26.9741) = 1.5 as shown in Figure 10. Therefore, the inflection point is (26.9741, 1.5).

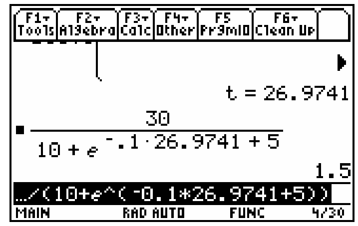


Figure 10

Example 5: 0.0559 and 1.788 are the only critical numbers for $f(x) = e^{5x}\ln(x/2)$. Determine if the critical point (0.0559, -4.73) is a relative minimum, a relative maximum or neither.

Solution 1: Recall that if $f''(0.0559)$ is positive the critical point is a relative minimum, if it is negative the point is a relative maximum, and if it is 0 the first derivative test must be used.

The key with the symbol | on it means "when" or "such that."

Now enter the following command:

d(e^(5x)*ln(x/2),x,2) | x=0.0559

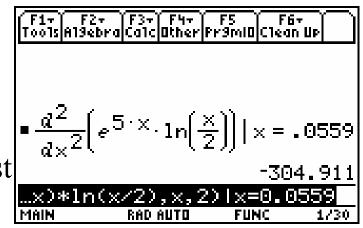


Figure 11

Since $f''(0.0559) = -304.911$ is negative as shown in Figure 11, the critical point is a relative maximum.

Solution 2: A graph that clearly showed the point (0.0559, -4.73) would reveal what was true for the critical point.

Press ◆F1 for the y= screen. Enter the function

y1=e^(5x)*ln(x/2)

as shown in Figure 12

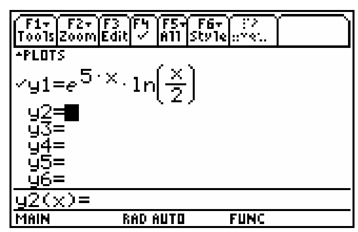


Figure 12

If F2 (Zoom) and choice 6:ZoomStd are selected (so that the x and y values go from -10 to 10), the result is Figure 13. Notice that this reveals nothing about the critical point in question. So it is desirable to look at the point more closely. (If you are "expert" at using ZoomIn, this approach can be used instead of the one shown. Just make sure the new center chosen has a y value near -4.73.)

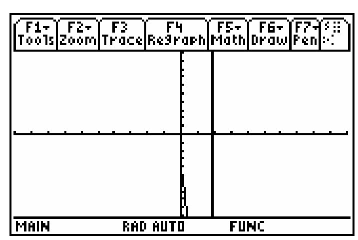


Figure 13

A good start might be to select values of x between -1 and 1 and values of y between -6 and -4. So press ◆F2 (Window) and enter the following values. xmin=-1 xmax=1 xscl=1 ymin=-6 ymax=-4 yscl=1 as shown in Figure 14.

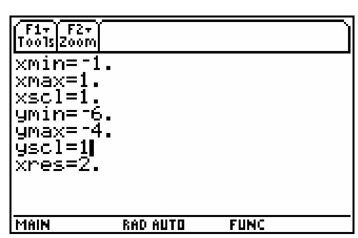


Figure 14

Then press ◆F3 (Graph). Figure 15 is the result.

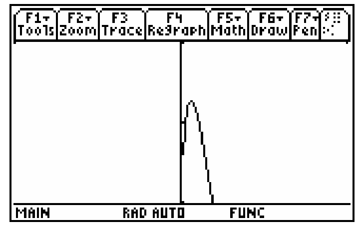


Figure 15

Now press F3 (Trace) and then press the right cursor arrow a few times and observe values of x and y for each point on the graph shown. Clearly the high point is the critical point and therefore the critical point is a relative maximum.

Students should also be familiar with using F5 (Math) in the graph window to determine the exact location of a relative maximum or relative minimum displayed on the graph. For the graph shown in Figure 15 press F5 (Math) to obtain the menu shown in Figure 16.
Select choice 4:Maximum.

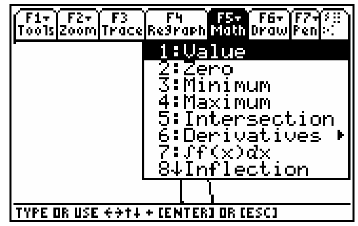


Figure 16

In response the "Lower Bound?" request, just use the cursor movement arrows to move the blinking cursor to the left of the maximum and press ENTER. Then, for the "Upper Bound?" request, move the blinking cursor to the right of the maximum and press ENTER. Figure 17 is the result and indicates that (0.05591, -4.73092) is a relative maximum.

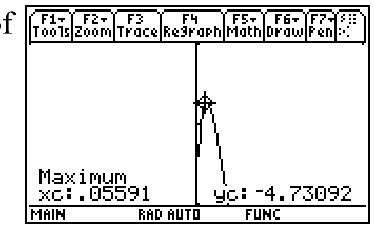


Figure 17

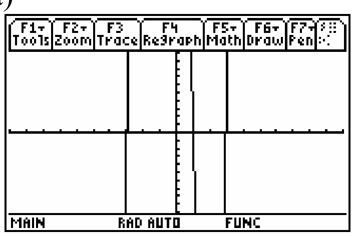
Example 6: Given $f(x) = x^4 + 14x^3 - 24x^2 - 126x + 135$

i) Find the critical numbers.

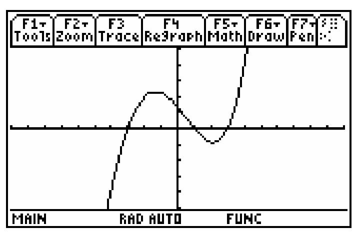
ii) Find the critical points

iii) All of the following are graphs of f(x) in different graphical windows. Which graph most accurately portrays the function (shows its relative extrema, asymptotes, etc.)?

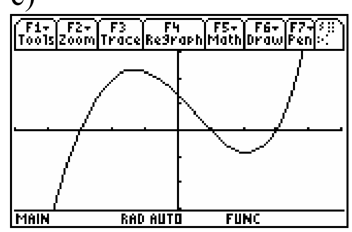
a)



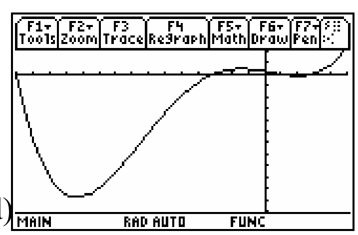
b)



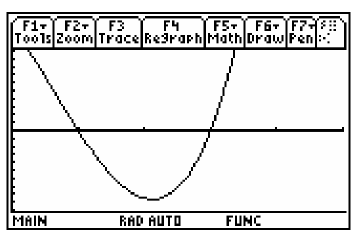
c)



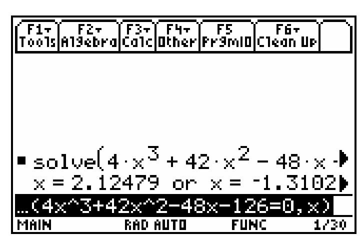
d)



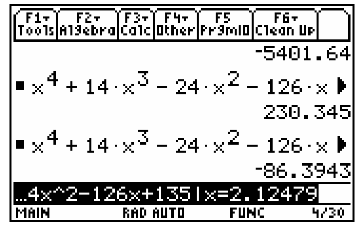
e)



Solution: i) The critical numbers are the solutions to
$0 = f'(x) = 4x^3 + 42x^2 - 48x - 126$.
Enter the command
solve(4x^3+42x^2-48x-126=0,x)
into the calculator as shown in the screen on the right.
The critical numbers are x = -11.3145, -1.31026 or 2.1247

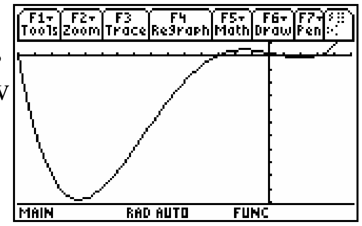


ii) For the critical points the value of y is needed for each of the critical numbers found.
Enter the command
x^4+14x^3-24x^2-126x+135 | x=-11.3145
into the calculator.
Repeat this for the other critical numbers. (Pressing the right cursor arrow removes the command line highlight and positions the cursor at the end of the line. Then the backspace key 🡐 can be used to eliminate the -11.3145. Then enter the next critical number.)



The screen shown indicates the critical points are (-11.3145, -5401.64), (-1.31026, 230.345) and (2.212479, -86.3943).

iii) A graphing screen should be used that includes the critical points found. The x values shown should include all values between -12 and 3. The y values shown should include all values between -5402 and 231. A reasonable window to choose would thus be
xmin=-15 xmax=5 xscl=1 ymin=-5500 ymax=500 yscl=500
The graph shown on the right is the result. So the answer is d.



Integration and the TI-89

Example 7: Evaluate $\int \frac{x}{x^2+1}dx$

In the HOME screen, Integration is under F3(calc)->2: $\int$ (integrate $\int(x/x^2+1),x)$

Figure 18 shows the entry on the TI-89.

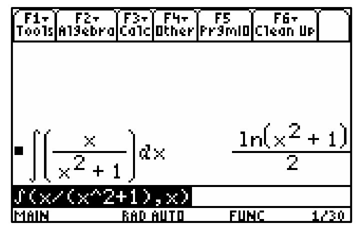


Figure 18

(Note that you must insert the constant of integration on your own.)

Example 8: Evaluate $\int_1^2 \frac{x}{x^2+1}dx$

The procedure is the same as for indefinite integrals, but with 2 extra arguments:
F3(calc)->2: $\int$ integration
$\int(x/x^2+1),x,1,2)$ (The last 2 arguments are the lower limit, followed by the upper limit.)
Figure 19 shows the entry on the TI-89.

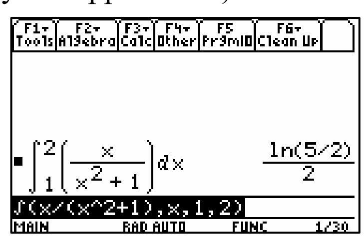


Figure 19

SAMPLE EXAM A

PART 1 - NO CALCULATORS PERMITTED

A1. Find all of the points at which the graph of $f(x) = x^4 - 4x + 5$ has horizontal tangent lines.

a) (2,0) only b) (1,0) only c) (1,2) and (-1,10) d) (1,0) and (2,13) e) (1,2) only

A2. If the derivative of g(x) is $g'(x) = (x - 1)^2(x - 2)$ then the function is

a) decreasing on $(-\infty, \infty)$
b) decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$
c) decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$
d) increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$
e) decreasing on $(-\infty, 1)$, increasing on $(1, 2)$ and decreasing on $(2, \infty)$

A3. The position equation for a moving body is $s = \dfrac{3t^2 - 4t + 10}{5}$ for $t \geq 0$.
Find the time when the velocity is 0.

a) 1 b) 3/2 c) 0 d) 5/6 e) 2/3

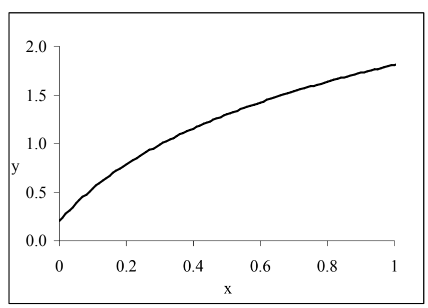
A4. The sum of a number and twice another number is 10. Find the largest possible product. (Notice that your answer is not one or both of the numbers, it is the product.)

a) 12.5 b) 8 c) 12 d) 12.6 e) 13

A5. Find the number of units to be produced in order to maximize revenue if the demand function is given by $p = 600x - 0.02x^2$.

a) 15,000 b) 20,000 c) 25,000 d) 30,000 e) 35,000

A6. On the interval $0 < x < 1$, for the graph shown,



a) $f'(x) > 0$ and $f''(x) > 0$
b) $f'(x) > 0$ and $f''(x) < 0$
c) $f'(x) < 0$ and $f''(x) > 0$
d) $f'(x) < 0$ and $f''(x) < 0$
e) None of the above

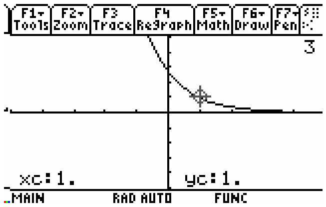
A7. Suppose $y = f(x)$ and $y = g(x)$ are inverse functions with $f(3) = 9$ and $f'(3) = 7$ then $g'(9)$ is

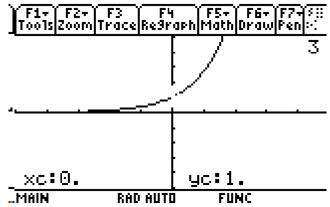
a) 7 b) -7 c) 1/7 d) -1/7 e) does not exist

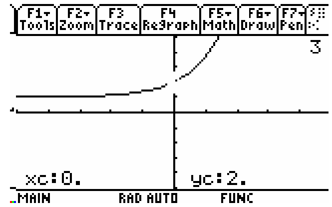
A8. A population of bacteria is decaying continuously at a rate of 7% every hour. If there are 3.2 million bacteria present at 9AM, which of the following gives the population of bacteria t hours after 9AM?

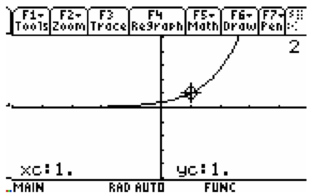
a) $P(t) = 3.2(1.07)^t$ b) $P(t) = 3.2(0.93)^t$ c) $P(t) = 3.2^t$
d) $P(t) = 3.2e^{-0.07t}$ e) $P(t) = 3.2e^{0.07t}$

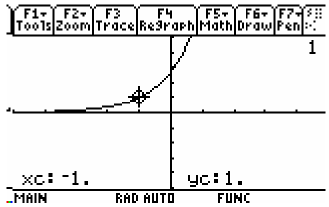
A9. Which of the following best represents the graph of $y = e^{x+1}$? (All *x*- and *y*- scales are 1.)

a) 

b) 

c) 

d) 

e) 

A10. Evaluate $\frac{d}{dx}\left(e^{-2x^2}\right)$

a) e^{-2x^2} b) $2x^2e^{2x^2-1}$ c) e^{-4x} d) $-4xe^{-2x^2}$ e) none of these

A11. Evaluate $log_3\sqrt{27}$

a) 1 b) 3/2 c) 2 d) 5/2 e) 9

A12. Evaluate $\frac{d}{dx}(ln2x)$

a) $\frac{1}{2x}$ b) $\frac{1}{xln2}$ c) $\frac{ln2}{x}$ d) $xln2$ e) $\frac{1}{x}$

A13. The marginal cost, in dollars is given by the equation $C'(x) = 2x + 200$. If the fixed overhead cost is \$2000, then the total cost of producing 10 items is

a) \$2000 b) \$2100 c) \$3500 d) \$4100 e) \$4500

A14. Evaluate $\int (2x-3)^4\,dx$

a) $\frac{(2x-3)^5}{5}+c$ b) $\frac{(2x-3)^3}{3}+c$ c) $\frac{(2x-3)^5}{10}+c$ d) $\frac{(2x-3)^3}{6}+c$ e) $\frac{(2x)^5}{5}-\frac{(3x)^5}{5}+c$

A15. The area of the region bounded by $f(x) = 3^x$, the lines $x = 1$ and $x = 2$ and the *x*-axis is to be approximated by 10 rectangles using the right endpoint of each rectangle. The sum is

a) $\frac{1}{10}\sum_{k=0}^{9} 3^{\frac{k}{10}}$ b) $\frac{1}{10}\sum_{k=1}^{10} 3^{\frac{k}{10}}$ c) $\frac{1}{10}\sum_{k=0}^{9} 3^{1+\frac{k}{10}}$ d) $\frac{1}{10}\sum_{k=1}^{10} 3^{1+\frac{k}{10}}$ e) none of these

A16. Evaluate $\int_0^4 \left(4x - 3\sqrt{x}\right)dx$.

a) 28 b) 32 c) 4 d) 16 e) 10

A17. Evaluate $\int_{-2}^{2} \frac{1}{3-x}dx$

a) 4/5 b) -ln5 c) ln5 d) 24/25 e) 1

A18. The consumer surplus for the demand and supply functions $x + y = 4$ and $-x + y = 2$ is

a) ½ b) 3/2 c) 5/2 d) 7/2 e) 4

A19. Find the critical numbers for $g(x) = x^4 - 2x^2$.

a) 0, $\pm\sqrt{2}$ b) 0, ±1 c) $\pm\sqrt{(3)}/3$ d) only 0, -1 e) only 0

A20. (0, 5) (i.e. $x = 0$ and $y = 5$) is the only critical point of $f(x) = 2x^6 + 3x^4 + 5$. You do not have to verify this. Determine what is true of (0, 5).

a) (0, 5) is a relative maximum. b) (0, 5) is a relative minimum c) (0, 5) is a saddle point
d) (0, 5) is not a relative extremum e) no conclusion is possible

A21. Find the maximum profit for the profit function $P(x) = -\frac{1}{2}x^2 + 6x$.

a) 18 b) 3 c) 1 d) 0 e) 12

A22. Let the profit function for a particular item be $P(x) = -10x^2 + 160x - 100$. Use differentials to approximate the increase in profit when production is increased from 5 to 6.

a) 30 b) 40 c) 55 d) 60 e) 70

A23. The equation of the tangent line to the curve defined by $f(x) = 2e^{2x^2-2} + 3x + 1$ at the point at which $x = 1$ is

a) $y = 4x + 2$ b) $y = 5x + 1$ c) $y = 11x - 5$ d) $y = -11x + 17$ e) $y = 17x - 11$

A24. $\frac{ln4 + ln3}{ln6} =$

a) ln 2 b) 2 c) $\frac{ln12}{ln6}$ d) ln 72 e) $\frac{ln4}{2}$

A25. $\int_0^1 x^2\sqrt[3]{8+19x^3}\,dx =$

a) 65/ 76 b) 64/25 c) 3 d) 4/3 e) ½

PART II - CALCULATORS PERMITTED

A26. Find the absolute extrema of $f(x) = \frac{2x}{x^2+1}$ on the closed interval [0, 2].

a) (0, 1) and (1, 3) b) (2, 4/5) and (-1, -1) c) (2, 4/5) and (0, 0)
d) (1, 1) and (-1, -1) e) (1, 1) and (0, 0)

A27. Find the relative maximum value for the function $f(x) = x^3 - 15x$

a) $-\sqrt{15} = -3.87$ b) $\sqrt{15} = 3.87$ c) $-10\sqrt{5} = -22.36$ d) $10\sqrt{5} = 22.36$ e) 12

A28. The profit from manufacturing x items is given by $P(x) = -0.5x^2 + 46x - 10$. Find ΔP on [20, 21].

a) \$25.00 b) \$25.50 c) \$26.00 d) \$710.00 e) \$735.50

A29. A job offer consists of a \$27,000 starting salary with a 4% increase each year. To the nearest dollar, what will the salary be in 7 years?

a) \$34,164 (b) \$35,530 (c) \$36,951 (d) \$38,429 (e) \$284,616

A30. The maximum value of $f(x) = xe^{-x}$ is

a) $e^{-1} \approx 0.368$ b) $e \approx 2.718$ c) 0.5 d) $-e \approx -2.718$ e) 0

A31. Given $\ln(2x - 3) = 4^{-x}$, then to three decimal places, $x =$

a) 1.540 b) 2.031 c) 3 .057 d) 4.201 e) 5.057

A32. If \$5,000 per year flows uniformly over an 8 year period and earns 3% interest, compounded continuously, then the present value, to the nearest dollar, is

a) \$454 b) \$35,562 c) \$45.208 d) \$55,116 e) \$73,205

A33. For the function $h(x) = \frac{x^2 e^x}{x}$, which of the following are true about the graph of $y = h(x)$?

I. The graph has a vertical asymptote at $x = 0$
II. The graph has a horizontal asymptote at $y = 0$
III. The graph has a minimum point.

a) None b) I and II only c) I and III only d) II and III only e) I, II and III

A34. The graph of $y = x^{\frac{1}{2}} - x^{\frac{3}{2}}$ is

a) concave down for $x > 0$ b) increasing for $x > 0$ c) has $y = 0$ as a horizontal asymptote

d) concave up for $x > 0$ e) has $x = 0$ as a vertical asymptote

A35. The "best fit" logarithmic function passing through the points (2, 1), (3, 4) and (4, 7), rounded to three decimal places is

a) $y = 3x - 5$
b) $y = 1.923 + 0.969 \ln x$
c) $y = -4.091 + 7.234 \ln x$
d) $y = 0.333x + 1.667$
e) $y = -5.083 + 8.574 \ln x$

SAMPLE EXAM B

PART 1 - NO CALCULATORS PERMITTED

B1. Find <u>all</u> of the critical numbers for $f(x) = 2\sqrt{9-x^2}$.

a) 0 b) -3, 3 c) 3 d) -3, 0, 3 e) -3

B2. Find the absolute maximum of $f(x) = 12x - x^3$ on the closed interval [-4, 1].

a) -4 b) -2 c) 16 d) 11 e) 0

B3. Solve $f''(x) = 0$ for the function $f(x) = 4x^3 - 9x + 1$.

a) $x = 4$ b) $x = 12$ c) $x = -9$ d) $x = 1$ e) $x = 0$

B4. Of all numbers (positive, negative and zero) whose difference is 4, find the two that have the minimum product. One of the numbers is:

a) 4 b) -3 c) -2 d) 1 e) -1

B5. Find the Elasticity of demand when the price is 4 for the demand function $px^2 = 100$

a) ½ b) 2 c) -2 d) -5/8 e) - ½

B6. The linearization of $f(x) = 5x^2 + 3x - 2$ near $x_0 = 1$ is:

a) $y = 10x + 13$ b) $y = 13x + 5$ c) $y = 13x - 7$ d) $10x + 3$ e) $10x - 4$

B7. $f(x) = 4x^3 + 2x^2 + 3x - 1$. Find $\left(f^{-1}\right)'(-6)$.

a) 11 b) 1/11 c) 411 d) 1/411 e) -1/6

B8. When writing the formula for an exponential function passing through the points (-2, 4/9) and (2, 36) using the form $y = ab^x$, what would be the base, b?

a) 4/9 b) 81 c) 1/81 d) 3 e) 1/3

B9. Which of the following represents the amount in an account t years after investing $3000 at a 6% annual rate compounded monthly?

a) $3000\left(\frac{0.06}{12}\right)^{12t}$ b) $3000(1+0.06)^{12t}$ c) $3000\left(1+\frac{0.06}{12}\right)^{12t}$

d) $3000\left(1+\frac{0.06}{12}\right)^{\frac{t}{12}}$ e) $3000e^{0.06(12t)}$

B10. If $f(x) = e^{x^2-3x}$, then $f'(x) =$

a) e^{x^2-3x} b) e^{2x-3} c) $e^{2x-3}(2x - 3)$ d) $e^{x^2-3x}(2x - 3)$ e) $e^{2x-3}(x^2 - 3x)$

B11. Evaluate $log_{\frac{1}{3}} 9^4$

a) 2 b) -2 c) 8 d) -8 e) 27/4

B12. Evaluate $(log 10)(log 20)$

a) log 10 + log 20 b) $2(log 10)^2$ c) 1 + log 2 d) $(log 4)(log 5)$ e) none of these

B13. The marginal cost (in thousands of dollars) of producing x hundred items is given by $C'(x) = 4$. If overhead is \$500, find the cost function, $C(x)$.

a) $C(x) = 504x + C_o$ b) $C(x) = 4x\text{-}500$ c) $C(x) = 2x^2 + 500x + C_0$

d) $C(x) = 4x + 500$ e) $C(x) = 496x$

B14. $\int \frac{x^2}{x^3+4} dx =$

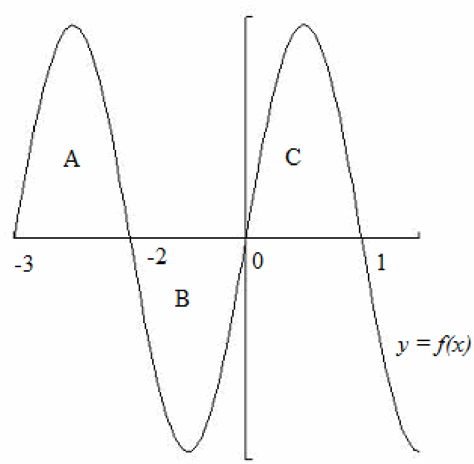
a) $x^2 ln|x^3+4| = c$ b) $\frac{x^3}{3} ln|x^3+4| + c$ c) $ln|x^3+4| + c$ d) $\frac{1}{3} ln|x^3+3| + c$ e) $e^{x^3+3} + c$

B15. Evaluate $\sum_{k=1}^{10} 3k$

a) 55 b) 165 c) 30 d) 150 e) 178

B16. Consider the function defined by $y = f(x)$, whose graph is given in the accompanying figure. Suppose the area of the region indicated by *A* is 5, the area of *B* is 3 and $\int_{-3}^{1} f(x)dx = 4$, then the area of the Region *C* is

a) 1 b) 2 c) 3 d) 4 e) none of these



B17. Evaluate $\int_{-1}^{0} (4x+1)^3 dx$

a) -80 b) -5 c) 7 d) -1/16 e) -1/4

B18. Find the APPROXIMATE area between the graph of $y = x^2$ and the x-axis between $x = 0$ and $x = 4$. For your approximation, use 4 rectangles of equal width and use the right-hand endpoint of each rectangle to determine the height of the rectangle.

a) 64/3 b) 14 c) 30 d) 21 e) 64

B19. Find any relative extrema of $f(x) = \dfrac{7x}{5x-3}$

a) $y = 0$ b) $y = 7/5$ c) $y = -7/3$ d) $y = 7$ e) there is no relative extremum

B20. Identify the x-coordinate of any relative extrema of $y = x\ln(x)$.

a) Relative min at $x = -1$ b) Relative max at $x = -1$ c) Relative min at $x = 1/e$
d) Relative max at $x = 1/e$ e) No relative extrema

B21. An apartment complex has 500 units available. At a monthly rent of \$1000 all the apartments are rented. For each \$10 increase in rent, 2 apartments become vacant. <u>If x is the number of \$10 increases in rent</u>, the monthly Revenue will be:

a) $(1000-10x)\cdot(500+2x)$ b) $(1000+2x)\cdot(500-10x)$ c) $(1000+10x)\cdot(500-2x)$

d) $(1000-2x)\cdot(500+10x)$ e) $(1000-500x)\cdot(10-2x)$

B22. If $y = x^2 + x$, find the value of the differential dy corresponding to a change in x of $dx = 0.1$ when $x = 2$.

a) 0.11 b) 0.5 c) 0.51 d) 1.2 e) 6.5

B23. Find dy/dx if $xe^y + 1 = xy$

a) 0 b) $\dfrac{y-e^y}{xe^y - x}$ c) $\dfrac{y}{e^y - x}$ d) $\dfrac{e^y}{xe^y - 1}$ e) $\dfrac{y}{xe^y - x}$

B24. $2^{log_2 5} =$

a) 5 b) 5/2 c) 2/5 d) -5 e) 5/4

B25. Find the critical numbers for $f(x) = x^3 + 3x^2 + 1$.

a) -2 only b) 0 and -2 c) 0 only d) $3x^2 + 6x$ e) There are no critical numbers.

PART II - CALCULATORS PERMITTED

B26. The absolute maximum value of the function $f(x) = x^3 - 6x^2 + 15$ on the closed interval [0, 3] is

a) -7 b) -17 c) 4 d) -12 e) 15

B27. What is the relative maximum of the function $y = f(x) = \frac{4x}{x^2+1}$?

a) (-1, 4) b) (1, 0) c) (0, 0) d) (1, 2) e) (2, 1.6)

B28. Given the total cost function $C(x) = 0.03x^2 + 32x + 4300$, find the value of x for which the average cost is a minimum.

(a) -533.333 (b) 533.333 (c) 378.594 (d) -378.594 (e) 500

B29. For the function $y = f(x) = x^4$, find Δy

a) $4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ b) $4x^3$ c) $4x^3dx$

d) $6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ e) $6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3$

B30. Find the inflection point for $g(t) = \frac{40}{5+2^{-t+4}}$

a) (4, 8) b) (1.678, 4) c) (0, 1.9048) d) (1.5546, 3.8290) e) (10.3782, 7.9808)

B31. The demand function for a product is $p = 100 - 0.1x \ln(x)$ where x is the number of units of the product sold and p is the price in dollars. Find the value of x for which the marginal revenue is one dollar per unit.

a) 97.45 b) 0.000017 c) 188.9 d) 190.5 only e) 0.01 and 190.5

B32. Find the best fit exponential function to three decimal places for the data given in the table below

x	1	2	3	4	5
y	27	30	34	35	40

a) $y = 26.388x^{0.230}$ b) $y = 24.479(1.103)^x$ c) $y = 0.071x^2 + 2.671x + 24.400$

d) $y = 3.100x + 23.900$ e) $y = 24.815(1.099)^x$

B33 When a certain radioactive element decays, the amount, in milligrams, that remains after t years can be approximated by the function $A(t) = ke^{-0.001t}$, where k is a constant. Approximately how many years would it take for an initial amount of 800 milligrams of this element to decay to 400 milligrams?

a) 173 b) 347 c) 693 d) 1386 e)2772

B34 If $4^{3x} = e^{kx}$, find k to 3 decimal places

a) 0.000 b) 4.159 c) 3.142 d) no solution e) 4.628

B35 Find the equation of the straight line that is tangent to $f(x) = xe^{x}$ at $x = 1$

a) $y = 5.437x - 8.155$ b) $y = 2.718x$ c) $y = 5.437x - 2.718$

d) $y = 2.718$ e) $y = 5.437x$

SAMPLE EXAM C

PART 1 - NO CALCULATORS PERMITTED

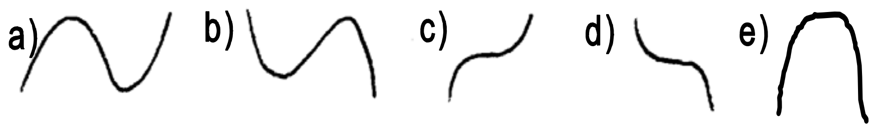
C1. The graph of the function $f(x)=\frac{x^2+2}{x^2+1}$ has as its vertical asymptote(s)

a) x = -1 only b) x = 1 only c) x = 1 and x = -1 d) No vertical asymptotes e) y = 1 only

C2. Find the open interval(s) on which $f(x)=\frac{x^2}{x^2+4}$ is decreasing.

a) (-∞, -2) and (2, ∞) b) (0, ∞) c) (-2, 2) d) (-∞, 0) e) (-∞, ∞)

C3. Given $y = 12x - x^3$, find the critical points. Then determine whether they are relative extrema (the second derivative test is easiest). Sketch the graph and pick the sketch that resembles your sketch the best.



C4 and C5. The height in feet above the ground of a ball thrown upwards from the top of a building is given by $s = -16t^2 + 160t + 200$, where t is the time in seconds

C4. What is the maximum height (in feet) of the ball?

a) 5 b) 600 c) 1400 d) 200 e) 160

C5. What is $v^{-1}(32)$?

a) 32 ft/sec b) 32 sec c) -864 ft/sec d) 4 ft/sec e) 4 sec

C6. For the function $y = f(x) = 3x^2 + 2x$, find dy.

a) $6x(\Delta x)+3(\Delta x)^2+2(\Delta x)$ b) $6x+2$ c) $(6x+2)dx$ d) $3(\Delta x)^2$ e) $3(\Delta x)$

C7. For a production level of x units of a commodity, the cost function in dollars is $C = 200x + 4100$. The demand equation is $p = 300 - 0.05x$. What price p will maximize the profit?

a) \$100 b) \$250 c) \$900 d) \$1500 e) \$6000

C8. An exponential graph containing the points (2, 5) and (4, 12) has the equation

a) $\frac{5}{3}(3^x)$ b) $\frac{3}{4}(2^x)$ c) $\frac{25}{12}\left(\frac{12}{5}\right)^{x/2}$ d) $\frac{12}{25}\left(\frac{5}{12}\right)^{x/2}$ e) none of these

C9. If $e^x = 243$ and $e^y = 32$ then $e^{\frac{3x+4y}{5}} =$

a) 6/5 b) 30 c) 864 d) 266 e) 432

C10. $\frac{d}{dx}\left(\left(e^x\right)^{10}\right) =$

a) $10e^{9x}$ b) e^{10x} c) $10e^{10x}$ d) $10\left(e^{x-1}\right)^{10}$ e) none of these

C11. Evaluate $ln\sqrt[7]{\frac{1}{e^5}}$

a) 7/5 b) $e^{-5/7}$ c) -5/7 d) $e^{7/5}$ e) none of these

C12. Evaluate $log\,\frac{x^3y^2}{\sqrt{y}z^5}$

a) $\frac{(logx^3)(logy^2)}{(log\sqrt{y})(logz^5)}$ b) $3logx + 2logy - \frac{1}{2}logy + 5logz$ c) $3logx + 2logy - \frac{1}{2}logy - 5logz$

d) $3logx + 2logy - 2logy + 5logz$ e) $3logx + 2logy - 2logy - 5logz$

C13. The acceleration of an object is given by the equation $a(t) = 5e^t + 2t - 1$. Determine its velocity function $v(t)$ if $v(0) = 6$.

a) $v(t) = -32t + 6$ b) $v(t) = 5e^t + \frac{t^3}{3} - \frac{t^2}{2} + t + 1$ c) $v(t) = 5e^t + t^2 - t + 11$

d) $v(t) = 5e^t + t^2 - t + 6$ e) $v(t) = 5e^t + t^2 - t + 1$

C14. Find the average value of $f(x) = x^3$ on the interval [0 , 2]

a) 2 b) 4 c) 1 d) 8 e) 3

C15. The area of the region bounded by $f(x) = 3x^2$, $g(x) = 4 - x$ and the x-axis from $x = ½$ to $x = 3$ is given by

a) $\int_{1/2}^{3} \left(3x^2 - (4-x)\right)dx$ b) $\int_{1/2}^{3} \left(4 - x - 3x^2\right)dx$ c) $\int_{1/2}^{1} 3x^2dx + \int_{1}^{3} (4-x)dx$

d) $\int_{1/2}^{1} (4-x)dx + \int_{1}^{3} 3x^2dx$ e) none of these

C16. Evaluate $\int_0^{\ln 3} e^{2x}dx$

a) $\frac{1}{2}(e^9 - 1)$ b) ½ c) 8 d) 4 e) 1

C17. The producer surplus for the demand and supply functions $x + y = 4$ and $-x + y = 2$ is

a) ½ b) 3/2 c) 5/2 d) 7/2 e) 3

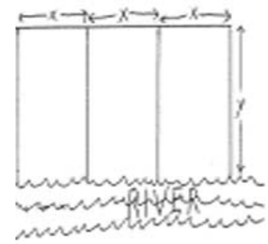
C18. The critical numbers for $y = x^4 - 2x^2$ are

a) $x = 0$ only b) $x = -1, 0, 1$ c) $x = -1, 1$ only d) $x = -2$ only e) $x = -4, 4$ only

C19. If $f(x) = 8x^4 - 10x^2 + 3x - 1$, find the value of $f''(x)$ (the second derivative) at $x = 2$.

a) 219 b) 364 c) 93 d) 376 e) 384

C20. A farmer wishes to construct 3 adjacent enclosures alongside a river as shown. Each enclosure is x feet wide and y feet long. No fence is required along the river, so each enclosure is fenced along 3 sides. The total enclosure area of all 3 enclosures combined is to be 900 square feet. What is the least amount of fence required?



a) 20 feet b) 120√2 feet c)120 feet d) 70√3 feet e) 360 feet

C21. Find the elasticity of demand when the price is 4 for the demand function $5p + 2x = 100$

a) -1/4 b) -5/4 c) -4 d) 1/4 e) 4

C22. For the function $y = f(x) = 3x^2 + 2x$, find Δy

a) $6x(\Delta x) + 3(\Delta x)^2 + 2(\Delta x)$ b) 6x + 2 c) $(6x + 2)dx$ d) $3(\Delta x)^2$ e) $3(\Delta x)$

C23. Find the derivative of $f(x) = 4e^{-3x}$.
a) $-12e^{3x}$ b) $12e^{-3x}$ c) $-12e^{-3x}$ d) $4e^{-3}$ e) $4e^{-3x}$

C24. If $3^{2x} = e^{kx}$ then $k =$

a) $3ln2$ b) $2ln3$ c) ln 3 - ln 2 d) $\frac{e}{3}$ e) $\frac{3}{e}$

C25. Use your knowledge of the definite integral (and your knowledge of the graph of the function $y = \sqrt{64 - x^2}$) in order to evaluate $\int_0^8 \sqrt{64 - x^2}\,dx$.

a) -8 b) 1024/3 c) 16π d) 32π e) 64π

PART II - CALCULATORS PERMITTED

C26. One critical number of the function $y = x^4 - 2x^2 - 3x + 1$ is

a) 1.263 b) -0.148 c) 0.752 d) 1 e) -1.172

C27. Find the relative maximum of $f(x) = 4\sqrt{x} - 2x + 1$ on the closed interval [0, 6].

a) 6 b) 3 c) $4\sqrt{6} - 11 \approx -1.20204$ d) -3 e) 0

C28. For the function $y = f(x) = x^4$, find ε

a) $4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ b) $4x^3$ c) $4x^3 dx$
d) $6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ e) $6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3$

C29. The half-life of a radioactive substance is 1500 years. How much of a 128 lbs. sample of this substance will remain after 9000 years?

a) 2 lbs b) 3 lbs c) 4lbs d) 5 lbs e) 64 lbs

C30. The function $f(x) = 8e^{-0.1x^2}$ has exactly two points of inflection. The value of x for one of these points of inflection is

a) 8 b) -16 c) -0.8 d) 0 e) $\sqrt{5} \approx 2.236$

C31. For which value of x does the graph of the function $f(x) = e^{-x} \ln x$ have a horizontal tangent? Round your answer to the nearest hundredth.

a) 1 b) 1.76 c) 2.81 d) 3.93 e) Never

C32. Given $\ln(2x - 3) = 4^{-x}$, then to three decimal places, $x =$

a) 1.540 b) 2.031 c) 3 .057 d) 4.201 e) 5.057

C33. How much should be deposited into an account today if it is to accumulate to $15000 in 7 years and it earns 3.8% annual interest compounded quarterly?

a) $11721.50 b) $11496.60 c) $11511.00 d) $19571.00 e) $9892.80

C34. Given $g(x) = \dfrac{x^2 - 16}{3x^4 + 4x^3}$, find $g''(-4)$

a) -1118 b) $-1/32 \approx -0.03125$ c) 0 d) $-1/64 \approx -0.015625$ e) $-13/1024 \approx -0.012695$

C35 If $20,000 per year flows uniformly over a 30 year period and earns 4.5% interest, compounded continuously, then the present value, to the nearest dollar, is

a) $5,185 b) $329,227 c) $275,384 d) $77,148 e) $487,139

SAMPLE EXAM D

PART 1 - NO CALCULATORS PERMITTED

D1. For what value(s) of x is the derivative of $y = f(x) = \dfrac{x^2}{x^2 - 4}$ equal to zero or undefined?

a) $x = 0$ only b) $x = 2$ only c) $x = -2$ only d) $x = -2, 2$ only e) $x = -2, 0, 2$

D2. The absolute maximum of the function $f(x) = x^2 - 9$ on the interval [-3, 3] is

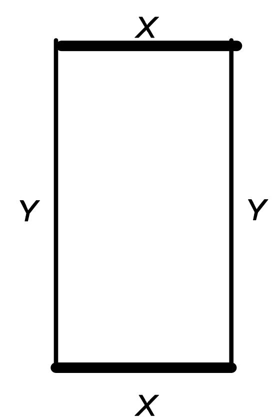
a) 0 b) 3 c) -3 d) 9 e) -9

D3. If $R = -x^3 + 3x^2 - 2$ is the revenue function, where x is the amount spent on advertising, then the point of diminishing returns (point of inflection) occurs when

a) $x = 1$ b) $x = -1$ c) $x = 6$ d) $x = -2$ e) Not enough information is given to decide

D4. Select the correct mathematical formulation of the following problem. A rectangular area must be enclosed as shown. The sides labeled x cost $20 per foot. The sides labeled y cost $10 per foot. If at most $300 can be spent, what should x and y be to produce the largest area?

a) Maximize xy if $40x + 20y = 300$
b) Maximize xy if $20x + 10y = 300$
c) Maximize xy if $2x + 2y = 300$
d) Maximize $2x + 2y$ if $xy = 300$
e) Maximize $2x + 2y$ if $200xy = 300$



D5. Find the values of x for which the profit $P(x) = x^3 - 12x^2 + 45x - 13$ is a maximum on [0, 5].

a) $x = 0$ b) $x = 3$ c) $x = 5$ d) $x = 3$ and 5 e) $x = 0$ and 5

D6. For the function $y = f(x) = 3x^2 + 2x$, find ε

a) $6x(\Delta x) + 3(\Delta x)^2 + 2(\Delta x)$ b) $6x + 2$ c) $(6x + 2)dx$ d) $3(\Delta x)^2$ e) $3(\Delta x)$

D7. Find the derivative of $f(x) = x^4e^x$

a) $4x^3e^x$ b) $x^4e^x + 4x^3e^x$ c) $4x^3 + e^x$ d) $x^4 + e^x$ e) $12x^4e^x$

D8. Given $\log(2 + x) + \log(x - 3) = \log 14$, then $x =$

a) -4 b) 5 c) 4 d) - 5 e) -4 and 5

D9. Solve: $\frac{dy}{dx} = \frac{3x^2}{y^4}$

a) $y = \sqrt{5x^3 + C}$ b) $y = \frac{5x^3}{y^5} + C$ c) $y = 5x^3 + C$

d) $y = \sqrt[5]{5x^3 + C}$ e) $y = \frac{3x(2y - 4x)}{y^5}$

D10. Evaluate $\int_{-3}^{3} (x^2 + 15)dx$

a) 54 b) 108 c) 0 d) 18 e) 36

D11. Find the area under the curve of $y = \sqrt{9 - x}$ on the interval [0, 9].

a) 18 b) -18 c) 3 d) 9 e) -9

D12. What is the absolute maximum value of $f(x) = x^3 - 75x$ on the closed interval [0, 4]?

a) 0 b) -250 c) 250 d) 236 e) 75

D13. Solve the equation $\left(\frac{1}{3}\right)^{x-1} = 27$

a) 2 b) there is no solution c) -3 d) -4 e) -2

D14. If the demand equation is given by $p = \sqrt{9 - x}$, then the marginal revenue is 0 when

a) $x = 9$ b) $x = 2$ c) $x = 6$ d) $x = 3$ e) $x = 5$

D15. The linearization of $f(x) = 5x^3 - 7x^2 + 11x - 1$ near $x_0 = 2$ is:

a) $y = 43x - 53$ b) $y = 15x + 3$ c) $y = -14x + 61$ d) $y = 5x + 23$ e) $y = -11x + 65$

D16. Find the equation of the line tangent to the graph of $y = xe^{2x - 4}$ at the point where $x = 2$.

a) $y = e^2x + 2$ b) $y = 2x - 2$ c) $y = 3x - 4$ d) $y = 4x - 6$ e) $y = 5x - 8$

D17. $\frac{ln40 - 3ln2}{\frac{1}{2}ln25} =$

a) 0 b) 5/4 c) 3 d) 1 e) 12/5

D18. Which of the following will give a result of $x^3 - 4x - 7$?

a) $\frac{d}{dx}\int_{-7}^{x}(3t^2 - 4)dt$ b) $\int_{0}^{x}(t^3 - 4t - 7)dt$ c) $\frac{d}{dt}\int_{2}^{x}(t^3 - 4t - 7)dt$

d) $\frac{d}{dx}\int_{2}^{x}(t^3 - 4t - 7)dt$ e) $\frac{d}{dx}(t^3 - 4t - 7)$

D19. Determine the *x*-coordinate of the point(s), if any, at which the graph of $f(x) = \frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{9}{2}x^2 + 11$ has a horizontal tangent.

a) x = -1, 9 b) None c) x = 11.778, 2.3601 d) x = 0, 1, 9 e) x = 11

D20. The position of an object at any time *t* is given by $s(t) = -8t^2 + 20t + 10$. Find the velocity when $t = 2$

a) 18 b) -12 c) -16 d) 22 e) 5/4

D21. Find the elasticity of demand when the price is 10 for the demand function $5x + 6p = 100$

a) - 2/3 b) - 3/2 c) 2/3 d) - 6/5 e) 3/2

D22. For the function $y = f(x) = x^4$, find dy

a) $4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ b) $4x^3$ c) $4x^3dx$

d) $6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4$ e) $6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3$

D23. Find the derivative of $f(x) = -6 \ln x$

a) $6/x$ b) $-6/x^2$ c) $-6/(\ln x)$ d) $6/x^2$ e) $-6/x$

D24 and D25. For the function $f(x) = x^3 - 9x^2 + 15x$

D24. Find the point(s) of inflection (x, y).
a) (1, 7) only b) (5, -25) only c) (1, 7) and (5, -25) d) (3, -9) e) (3, -12)

D25. Find the **absolute** extrema on the closed interval [0, 3].

a) minimum is -9 and maximum is 0
b) minimum is -9 and maximum is 7
c) minimum is -25 and maximum is 7
d) minimum is -25 and maximum is 0
e) minimum is 0 and maximum is 7

PART II - CALCULATORS PERMITTED

D26. Given $f(x) = \dfrac{x^3 - 1}{x^4 + 5x + 17}$ find $f''(1)$.

a) 84 b) 0 c) 529 d) 84/529 e) 1

D27 and D28. The weekly profit that results from selling x units of a commodity weekly is given by $P = 50x - 0.003x^2 - 5000$ dollars.

D27. Find the actual weekly change in profit that results from increasing production from 5000 units weekly to 5015 units weekly.

a) \$20.00 b) \$300.00 c) \$299.33 d) \$298.00 e) \$8333.33

D28. Use the marginal profit function to estimate the change in profit that results from increasing production from 5000 units weekly to 5015 units weekly.

a) \$20.00 b) \$300.00 c) \$299.33 d) \$298.00 e) \$8333.33

D29. $x = -1/3 \approx -0.333$ is a critical number for $f(x) = e^x \sqrt[3]{x}$. The critical point (-0.333, -0.497) is

a) a relative minimum b) a relative maximum c) neither a relative maximum nor minimum
d) a point of inflection e) a saddle point

D30. The "best fit" logarithmic function passing through the points (2, 1), (3, 4) and (4, 7), rounded to three decimal places is

a) $-5.083 + 8.574 \ln x$ b) $8.574 - 5.083 \ln x$ c) $3.623 + 2.57 \ln x$

d) $2.57 + 3.623 \ln x$ e) none of these

D31. Find the critical numbers for $f(x) = e^{10x} \ln(x)$.

a) 0.894 only b) 0.028 and 0.894 c) 0.789 only d) 0.789 and 0.894 e) There are no critical numbers

D32. Find the equation of the straight line that is tangent to $f(x) = 2^x$ at the point (5, 32).

a) $y = 2x + 22$ b) $y = 22.1807x - 78.9035$ c) $y = 15.3745x - 44.8725$
d) $y = 32x - 128$ e) $y = 21.5678x - 75.839$

D33. Given the demand and supply functions $p = 2 - x^2$ and $p = 1 + x^2$ determine the producer surplus to three decimal places.

a) 0.825 b) 0.236 c) 1.531 d) 1.061 e) 1.296

D34 Determine the absolute extrema of the function $f(x) = x\sqrt[4]{(1-x)}$ on the interval [0 , 1]

a) 0 and 1

b) 0 and 0.535

c) there is no absolute maximum

d) 0 and 0.122

e) there are no absolute extrema

D35 Given that the decay constant for Radium is – 0.000428/year, how long, to the nearest year, does it take a sample to decay to 15% of its present mass?

a) 4433

b) 5120

c) 3827

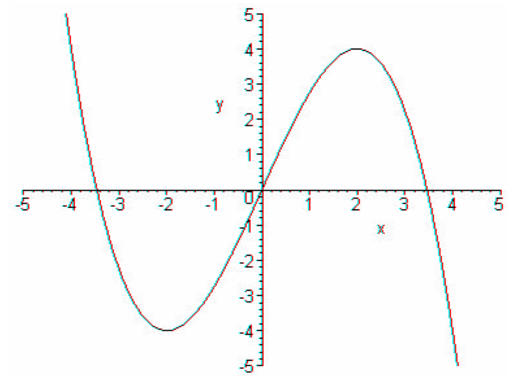
d) 27815

e) 53142

SAMPLE EXAM E

PART 1 - NO CALCULATORS PERMITTED

E1. The graph of $y = f(x)$ appears on the right. Estimate the points at which the absolute minimum and the absolute maximum occur on the interval [0, 3], that is, $0 \le x \le 3$.



a) absolute minimum: (0, 0); absolute maximum: (3, 2)
b) absolute minimum: (0, 0); absolute maximum: (2, 4)
c) absolute minimum: (-2, -4); absolute maximum: (2, 4)
d) absolute minimum: (4, -5); absolute maximum: (-4, 5)
e) absolute minimum: (3, 2); absolute maximum: (2, 4)

E2. Find the open interval(s) on which $f(x) = x^3 - 3x + 7$ is increasing.

a) $(-\infty, -1)$ or $(1, \infty)$ (i.e. $x < -1$ or $x > 1$)
b) $(1, \infty)$ only (i.e $x > 1$ only)
c) $(-1, 1)$ (i.e. $-1 < x < 1$)
d) $(-\infty, -1)$ only (i.e. $x < -1$ only)
e) $(-\infty, \infty)$ (i.e. all real x)

E3. The position of an object at any time t is given by $s(t) = -8t^2 + 20t + 10$. Find the acceleration when $t = 2$

a) 18 b) -12 c) -16 d) 22 e) 5/4

E4. The position of an object at any time t is given by $s(t) = -8t^2 + 20t + 10$. Find the time when the velocity is 0.

a) 18 b) -12 c) -16 d) 22 e) 5/4

E5. Find the maximum profit for the profit function $P(x) = -2x^2 + 10x - 3$.

a) 10 b) 19/2 c) $(5 + \sqrt{19})/2$ d) 7/4 e) 67/8

E6. A square is measured and each side is found to be 5 inches with a possible error of at most .03 inches. Use DIFFERENTIALS to find the approximate error in computing the area of the square.

a) .6 b) .06 c) .006 d) .3 e) .03

E7. Find the derivative of $f(x) = 3e^{-5x + 7}$.

a) $-15e^{-5}$ b) $-15e^{-5x + 7}$ c) $3e^{-5}$ d) $-15e^{-5x + 7} + 3e^{-5x + 7}$ e) $3e^{-5} + 3e^{-5x + 7}$

E8. Evaluate $e^{3\ln 5}$

a) 15 b) $12e$ c) 125 d) 3/5 e) 243

E9. Find the derivative of $f(x) = \ln\left(\frac{x^2-7}{x}\right)$.

a) $\frac{1}{x^2-7} - \frac{1}{x} = \frac{-x^2+x+7}{x(x^2-7)}$ b) $\frac{1}{x^2-7} + \frac{1}{x} = \frac{x^2+x-7}{x(x^2-7)}$ c) $\frac{2x}{x^2-7} + \frac{1}{x} = \frac{3x^2-7}{x(x^2-7)}$

d) $\frac{2x}{x^2-7} - \frac{1}{x} = \frac{x^2+7}{x(x^2-7)}$ e) $\frac{x}{x^2-7} + \frac{1}{x} = \frac{2x^2-7}{x(x^2-7)}$

E10. Solve $y' = 5x^3y^2$.

a) $y = \frac{-4}{5x^4+4C}$ b) $y = \frac{4}{5x^4+4C}$ c) $y = \frac{5x^4y^3}{12} + C$

d) $y = \frac{-5}{4}x^4 + C$ e) $y = \frac{5}{4}x^4 + C$

E11. Evaluate $\int_4^8 \frac{1}{\sqrt{2t-4}}\,dt$.

a) 4 b) 2 c) $\frac{1}{\sqrt{10}} - \frac{1}{2}$ d) $2\sqrt{3} - 2$ e) $2\sqrt{3} + 2$

E12 and E13. For the function $f(x) = 4x - x^2 - 2$

E12. Find all critical points (x, y).

a) (2, 2) b) (2, -2) c) (2, 0) d) (-2, -14) e) (0, -2)

E13. Find the **absolute** extrema on the closed interval [0, 5] (i.e $0 \le x \le 5$).

a) minimum is -2 and maximum is 2 b) minimum is -7 and maximum is -2
c) minimum is -7 and maximum is 2 d) minimum is -7 and maximum is 5
e) minimum is -10 and maximum is 10

E14. The demand function and cost function for x units of a product are $p = \frac{60}{\sqrt{x}}$ and $C = 0.65x + 400$.

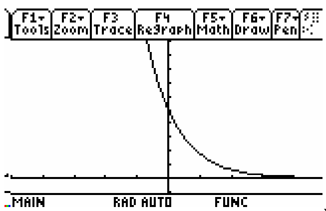
Find the marginal profit when $x = 100$.

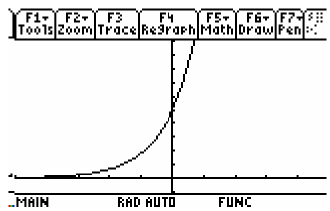
a) $2.35 per unit b) $4.58 per unit c) $193.50 per unit d) $187.35 per unit e) $3.65 per unit

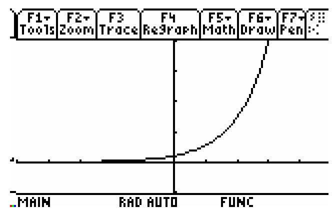
E15. If $f(12)=200$ and $f'(12)=-6$, estimate the value of $f(14)$ for the function $y=f(x)$

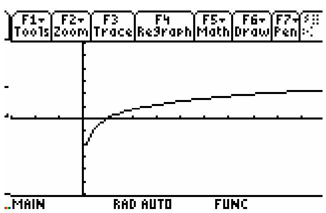
a) 194 b) 188 c) 206 d) 214 e) 8

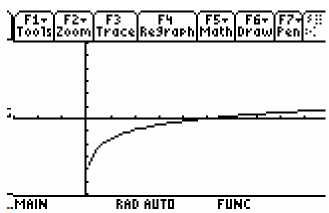
E16. Given the function $f(x)=5e^x$, which of the following would be the graph of $f^{-1}(x)$?
(All x- and y-scales are 1.)

a) 

b) 

c) 

d) 

e) 

E17. $8e^{ln3-ln2}=$

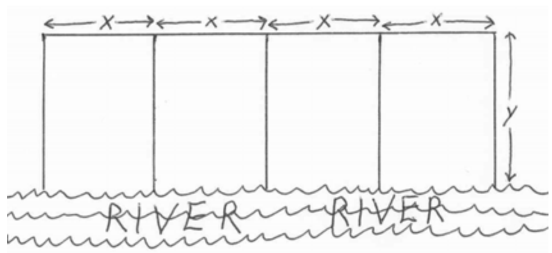
a) 8 b) $\frac{e^{3/2}}{8}$ c) 48 d) 18 e) 12

E18. Evaluate $\frac{d}{dx}\int_{-2}^{x}\ln t\,dt$

a) $\ln x$ b) $\frac{1}{x}$ c) $\frac{1}{t}$ d) $\frac{1}{x}+\frac{1}{2}$ e) $\ln x-\ln 2$

E19. Select the correct mathematical formulation of the following problem.
A farmer wishes to construct 4 adjacent fields alongside a river as shown. Each field is x feet wide and y feet long. No fence is required along the river, so each field is fenced along 3 sides. The total area enclosed by all 4 fields combined is to be 800 square feet. What is the least amount of fence required?

(a) Minimize xy if $4x + 5y = 800$
(b) Minimize xy if $4x + 5y = 200$
(c) Minimize $4x + 5y$ if $xy = 800$
(d) Minimize $4x + 5y$ if $xy = 200$
(e) Minimize $4(x + y)$ if $xy = 800$



E20. If $x^2 + y^2 = 24$ find $\frac{d^2y}{dx^2}$

a) $\frac{-x}{y}$ b) $\frac{-1}{y}$ c) $\frac{-24}{y^3}$ d) -2 e) -1

E21. Find the number of units that will minimize the average cost function if the total cost function is
$C = \frac{x^2}{4} - 3x + 400$

a) 6 b) 10 c) 20 d) 40 e) 80

E22. Find the derivative of $y = \ln(2x+3)^7$

a) $14\ln(2x+3)^6$ b) $14\ln(2x+3)^7$ c) $\frac{14(2x+3)^6}{\ln(2x+3)^7}$ d) $\frac{14}{2x+3}$ e) $\frac{7}{2x+3}$

E23. The position of an object at any time t is given by $s(t) = 2t^3 - 54t + 110$. Find the acceleration when $t = 2$

a) -30 b) 3 c) 18 d) 24 e) ½

E24. Find the average value of $f(x) = x^2 + x$ on the interval [0, 1]

a) 7/6 b) 2/3 c) 1 d) 1/2 e) 5/6

E25. When expressed as a single logarithm, $3\log(x-1) - \frac{1}{2}\log(y+1) - 2\log x + \frac{1}{4}\log z =$

a) $\log\frac{(x-1)^3\sqrt{y+1}}{x^2\sqrt[4]{z}}$ b) $\log\frac{(x-1)^3\sqrt{y+1}\sqrt[4]{z}}{x^2}$ c) $\log\frac{(x-1)^3\sqrt[4]{z}}{x^2\sqrt{y+1}}$

d) $\log(x-1)^3\sqrt[4]{z}x^2\sqrt{y+1}$ e) none of these

PART II – CALCULATORS PERMITTED

E26. \$1200 is invested for 10 years at an interest rate of $7\frac{1}{2}\%$ compounded MONTHLY. Determine, to the nearest dollar, the total amount that accumulates.

a) \$2540 b) \$2534 c) \$2473 d) \$2408 e) \$2280

E27. Evaluate $\lim_{x\to 0} \frac{x^2}{(e^x - 1)^2}$

a) 1 b) 0 c) ∞ d) undefined e) e

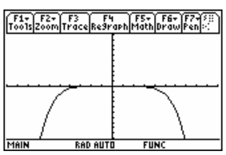
E28 and E29. Given $f(x) = \frac{e^x - x^6}{10{,}000}$

E28. Find the critical numbers.

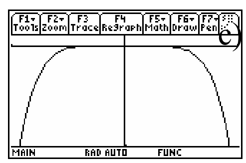
a) 0.482 only b) 0.482 and 13.94 c) 1 d) 0.824 only e) 0.824 and 15.494

E29. The graphs shown below display different graphical windows. More than one may actually be graphs the function f(x) shown above. Pick the graph that most accurately portrays the function (shows all its relative extrema, asymptotes, etc.).

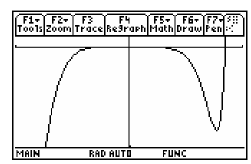
a)



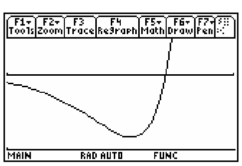
b)



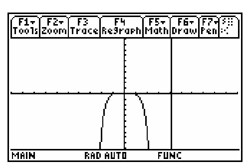
c)



d)



e)



E30. To three decimal places, $log_3 12 =$

a) 1.573 b) 2.262 c) 3.145 d) 4.879 e) 5.741

E31. If $f(x) = \frac{12ln(3x^2 + 2e^{4x^2+7})}{\sqrt{3x^2+1}}$ then to two decimal places, $f'(1) =$

a) -3.67 b) 2.69 c) 12.86 d) -4.62 e) none of these

E32. If x dollars are spent on advertising, then the revenue in dollars is given by $R = \dfrac{100{,}000}{1+2^{15-0.005x}}$ where $0 \le x \le 5000$. How much is spent on advertising at the point of diminishing returns (point of inflection)?

a) \$10,576 b) \$100,000 c) \$50,086.60 d) \$3000

e) There is no point of diminishing returns

E33. The area of the region bounded by $f(x) = x^2$, the x-axis and the line $x = 1$ and $x = 5$ is approximated by $n = 4$ rectangles. If the right endpoint is used for each of these rectangles, then the approximate area obtained is

a)30 b) 41 c) 42 d) 54 e) 64

E34 Given $g(x) = \dfrac{x^5 + 3x^3 + 4x^2 + 1}{x^4 - x^2 + 2}$, find $g''(0.5)$

a) 3.98692 b) 1297.5 c) -122.438 d) 0 e) 12.2258

E35 Linearize $f(x) = (x^3 + 2x + 1)^{1.7}$ near $x = 0$

a) $3.4x + 1$ b) $1.7x + 1$ c) $2x + 1$ d) $1.7x$ e) 1

ANSWERS

A1 e	B1 d	C1 d	D1 e	E1 b
A2 c	B2 c	C2 d	D2 a	E2 a
A3 e	B3 e	C3 b	D3 a	E3 c
A4 a	B4 c	C4 b	D4 a	E4 e
A5 b	B5 e	C5 e	D5 b	E5 b
A6 b	B6 c	C6 c	D6 e	E6 d
A7 c	B7 b	C7 b	D7 b	E7 b
A8 d	B8 d	C8 c	D8 b	E8 c
A9 e	B9 c	C9 e	D9 d	E9 d
A10 d	B10 d	C10 c	D10 b	E10 a
A11 b	B11 d	C11 c	D11 a	E11 d
A12 e	B12 c	C12 c	D12 a	E12 a
A13 d	B13 d	C13 e	D13 e	E13 c
A14 c	B14 d	C14 a	D14 c	E14 a
A15 d	B15 b	C15 c	D15 a	E15 b
A16 d	B16 b	C16 d	D16 e	E16 e
A17 c	B17 b	C17 a	D17 d	E17 e
A18 a	B18 c	C18 b	D18 d	E18 a
A18 b	B19 e	C19 b	D19 d	E19 d
A20 b	B20 c	C20 c	D20 b	E20 c
A21 b	B21 c	C21 a	D21 b	E21 d
A22 a	B22 b	C22 a	D22 c	E22 d
A23 c	B23 b	C23 c	D23 e	E23 d
A24 c	B24 a	C24 b	D24 d	E24 e
A25 a	B25 b	C25 c	D25 b	E25 c
A26 e	B26 e	C26 a	D26 d	E26 b
A27 d	B27 d	C27 b	D27 c	E27 a
A28 b	B28 c	C28 e	D28 b	E28 e
A29 b	B29 a	C29 a	D29 a	E29 c
A30 a	B30 b	C30 e	D30 a	E30 b
A31 b	B31 a	C31 b	D31 b	E31 d
A32 b	B32 e	C32 b	D32 b	E32 d
A33 d	B33 c	C33 c	D33 b	E33 d
A34 a	B34 b	C34 b	D34 b	E34 e
A35 e	B35 c	C35 b	D35 a	E35 a