MTH 4125 – Fall 2022
Course Syllabus

Course	Introduction to Stochastic Processes
Professor	C Douglas Howard
Office Hours	Tuesdays and Thursdays 1:55pm – 2:55pm. Tuesdays are virtual, Thursdays VC 6227.
Email	Douglas.Howard@baruch.cuny.edu
Text	Elements of Stochastic Processes: A Computational Approach, C Douglas Howard, FE Press, 2017
Grading	Based on 2 in-class exams (2/3) and a final exam (1/3)
Class Meets	Tuesdays (virtual) and Thursdays (VC 10-140) 2:55 - 4:35 pm

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Topics

Preliminaries
- Quick review of probability and expectation
- Discrete vs uncountable outcome spaces
- The unit interval $\mathbf{\Omega} = (0, 1)$ as outcome space
- The inverse transform construction and IID sequences on $\mathbf{\Omega}$

Some classic limit theorems for IID sequences
- Convergence in probability vs almost sure convergence
- Chebyshev's inequality and the weak law of large numbers
- The Borel-Cantelli lemma and the strong law of large numbers

Markov chains
- Conditional independence and the Markov property
- Basic properties of states
- Communication classes and their properties
- Ergodicity and invariant measures
- Example: an interacting particle system
- When the Markov chain is not ergodic

Random walks
- Gambler's ruin
- Recurrence, but lack of positive recurrence, in one dimension
- The reflection principle
- Recurrence in two dimensions
- Transience in three or more dimensions
- Stirling's formula

Arrival processes

- Arrival times, inter-arrival times, and the counting process
- The homogeneous Poisson process
- The inhomogeneous Poisson process
- Point processes in two and three dimensions
- When inter-arrival times are IID but not exponential

Brownian motion and related processes

- Definition and properties
- The Lévy construction
- Introduction to stochastic integration
- Itô's formula
- The Ornstein-Uhlenbeck process
- Applications from finance and economics

MTH 4125

LEARNING GOALS OF COURSE: Upon completion of this course, students will be able to:

- Understand the unit interval as a probability space and know how to generate a realization of a sequence of IID random variables when given a number chosen randomly from the unit interval.
- Know the difference between convergence in probability and almost sure convergence as applied to the Weak and Strong Laws of Large Numbers.
- Understand the property of Markov and identify Markovian processes.
- Diagram a Markov chain (MC) given its probability transition matrix.
- Identify the communication classes of a MC and determine which are recurrent and which are transient.
- Calculate the periods of a MC's recurrent communication classes.
- Understand ergodicity and be able to identify ergodic and irreducible MCs and compute their unique invariant distribution.
- Classify all invariant distributions for reducible MCs.
- Solve the Gambler's Ruin problem for random walks (RWs) on the integers and use it to show that the symmetric RW is recurrent while an asymmetric RW is transient.
- Solve expected return time problems for various RWs.
- Understand the reflection principle and use it to solve problems.
- State the various equivalent characterizations of a homogeneous Poisson process and use them to construct such processes.
- Use the inhomogeneous Poisson process to model arrival times (insurance claims, e.g.).
- Calculate long-term average waiting times for certain arrival processes that are not Poisson.
- Understand the defining properties of Brownian motion and use them to execute simple calculations.